TABLE 5.3.2.1-1: VALUES OF ϕ OR z ($t'_{1-\alpha}$) MOST COMMONLY USED IN MAINTAINABILITY ANALYSIS

$1-\alpha$	ϕ or z ($t_{1-\alpha}$)
0.80	0.8416
0.85	1.036
0.90	1.282
0.95	1.645
0.99	2.326

Following is an example of maintainability analysis of a system which has a lognormal distribution of repair times.

5.3.2.1.1 GROUND ELECTRONIC SYSTEM MAINTAINABILITY ANALYSIS EXAMPLE

Given the active repair times data of Table 5.3.2.1.1-1 on a ground electronic system find the following:

1. The probability density function, g(t)
2. The MTTR of the system
3. The median time to repair the system
4. The maintainability function
5. The maintainability for a 20 hour mission
6. The time within which 90% and 95% of the maintenance actions are completed.
7. The repair rate, u(t), at 20 hours.

TABLE 5.3.2.1.1-1: TIME TO REPAIR DATA ON A GROUND ELECTRONIC SYSTEM

Group No.	Times to repair t, hr	Frequency of observation n
1	0.2	1
2	0.3	1
3	0.5	4
4	0.6	2
5	0.7	3
6	0.8	2
7	1.0	4
8	1.1	1
9	1.3	1
10	1.5	4
11	2.0	2
12	2.2	1
13	2.5	1
14	2.7	1
15	3.0	2
16	3.3	2
17	4.0	2
18	4.5	1
19	4.7	1
20	5.0	1
21	5.4	1
22	5.5	1
23	7.0	1
24	7.5	1
25	8.8	1
26	9.0	1
27	10.3	1
28	22.0	1
$N' = 29$	24.5	1

1. Probability Density Function of g(t)

To determine the lognormal pdf of the times-to-repair given in Table 5.3.2.1.1-1, the values of $\overline{t'}$ and $\sigma_{t'}$, should be calculated from

$$\overline{t'} = \frac{\sum_{j=1}^{N'} n_j \, t'_j}{\sum_{j=1}^{N'} n_j} \tag{5.79}$$

where n_j is the number of identical observations given in the third column of Table 5.3.2.1.1-1, N' is the number of different-in-value observed times-to-repair, or number of data groups, which for this problem is N = 29, given in the second column of Table 5.3.2.1.1-1, and N is the total number of observed times-to-repair,

$$N = \sum_{i=1}^{N'} n_j$$

which, for this example, is 46.

And

$$\sigma_{t'} = \left[\frac{\sum_{i=1}^{N} (t'_i)^2 - N(\overline{t'})^2}{N-1}\right]^{\frac{1}{2}} = \left[\frac{\sum_{j=1}^{N'} n_j(t'_j)^2 - N(\overline{t'})^2}{N-1}\right]^{\frac{1}{2}} \tag{5.80}$$

To facilitate the calculations, Table 5.3.2.1.1-2 was prepared. From Table 5.3.2.1.1-2, $\overline{t'}$ and $\sigma_{t'}$, are obtained as follows:

$$\overline{t'} = \frac{\sum_{j=1}^{N'} n_j t'_j}{\sum_{j=1}^{N'} n_j} = \frac{30.30439}{46}$$

or

$$\overline{t'} = 0.65879$$

TABLE 5.3.2.1.1-2: CALCULATIONS TO DETERMINE $\bar{t}'$ and σ_t FOR THE DATA IN TABLE 5.3.2.1.1-1

t	$\log_e t = t'$	$(t')^2$	n	nt'	$n(t')^2$
0.2	-1.60944	2.59029	1	-1.60944	2.59029
0.3	-1.20397	1.44955	1	-1.20397	1.44955
0.5	-0.69315	0.48045	4	-2.77260	1.92180
0.6	-0.51083	0.26094	2	-1.02166	0.52188
0.7	-0.35667	0.12721	3	-1.07001	0.38163
0.8	-0.22314	0.04979	2	-0.44628	0.09958
1.0	0.00000	0.00000	4	0.00000	0.00000
1.1	0.09531	0.00908	1	0.09531	0.00908
1.3	0.26236	0.06884	1	0.26236	0.06884
1.5	0.40547	0.16440	4	1.62188	0.65760
2.0	0.69315	0.48045	2	1.38630	0.96090
2.2	0.78846	0.62167	1	0.78846	0.62167
2.5	0.91629	0.83959	1	0.91629	0.83959
2.7	0.99325	0.98655	1	0.99325	0.98655
3.0	1.09861	1.20695	2	2.19722	2.41390
3.3	1.19392	1.42545	2	2.38784	2.85090
4.0	1.38629	1.92181	2	2.77258	3.84362
4.5	1.50408	2.26225	1	1.50408	2.26225
4.7	1.54756	2.39495	1	1.54756	2.39495
5.0	1.60994	2.59029	1	1.60994	2.59029
5.4	1.68640	2.84394	1	1.68640	2.84394
5.5	1.70475	2.90617	1	1.70475	2.90617
7.0	1.94591	3.78657	1	1.94591	3.78657
7.5	2.01490	4.05983	1	2.01490	4.05983
8.8	2.17475	4.72955	1	2.17475	4.72955
9.0	2.19722	4.82780	1	2.19722	4.82780
10.3	2.33214	5.43890	1	2.33214	5.43890
22.0	3.09104	9.55454	1	3.09104	9.55454
24.5	3.19867	10.23151	1	3.19867	10.23151

$$\sum_{j=1}^{N'=29} n_j = 46 = N \qquad \sum_{j=1}^{N'} n_j t'_j = 30.30439 \qquad \sum_{j=1}^{N'} n_j (t'_j)^2 = 75.84371$$

and from Eq. (5.80)

$$\sigma_{t'} = \left[\frac{75.84371 - 46\ (0.65879)^2}{46-1} \right]^{\frac{1}{2}}$$

or

$$\sigma_{t'} = 1.11435$$

Consequently, the lognormal pdf representing the data in Table 5.3.2.1.1-1 is

$$g(t) = \frac{1}{t\ \sigma_{t'}\ \sqrt{2\pi}}\ e^{-\frac{1}{2}\left(\frac{t' - \bar{t'}}{\sigma_{t'}}\right)^2}$$

or

$$g(t) = \frac{1}{t(1.11435)\ \sqrt{2\pi}}\ e^{-\frac{1}{2}\left(\frac{t' - 0.65879}{1.11435}\right)^2}$$

where $t' = \log_e t$. The plot of this pdf is given in Figure 5.3.2.1.1-1 in terms of the straight times in hours. See Table 5.3.2.1.1-3 for the g(t) values used.

The pdf of the $\log_e t$ or of the t's is

$$g(t') = \frac{t}{t\sigma_{t'}\ \sqrt{2\pi}}\ e^{-\frac{1}{2}\left(\frac{t' - \bar{t'}}{\sigma_{t'}}\right)^2} = t\ g(t)$$

or

$$g(t') = \frac{1}{(1.11435)\sqrt{2\pi}}\ e^{-\frac{1}{2}\left(\frac{t' - 0.65879}{1.11435}\right)^2}$$

This pdf is that of a normal distribution which is what one should expect since if t follows a lognormal distribution, $\log_e t$ should be normally distributed. This is shown plotted in Figure 5.3.2.1.1-2, the values of g(t') were obtained from Table 5.3.2.1.1-3.

TABLE 5.3.2.1.1-3: The probability density of Time to Repair Data (From Table 5.3.2.1.1-1 based on the straight times to repair and the natural logarithm of the times to repair used to plot Figures 5.3.2.1.1-1 and 5.3.2.1.1-2, respectively.*)

Time to restore, t hours	Probability density, g(t)	Probability density $g(t') = g(\log_e t)$
0.02	0.00398	7.95×10^{-5}
0.1	0.10480	0.01048
0.2	0.22552	0.04510
0.3	0.29510	0.08853
0.5	0.34300	0.17150
0.7	0.33770	0.23636
1.0	0.30060	0.30060
1.4	0.24524	0.34334
1.8	0.19849	0.35728
2.0	0.17892	0.35784
2.4	0.14638	0.35130
3.0	0.11039	0.33118
3.4	0.09260	0.31483
4.0	0.07232	0.28929
4.4	0.06195	0.27258
5.0	0.04976	0.24880
6.0	0.03559	0.21351
7.0	0.02625	0.18373
8.0	0.01985	0.15884
9.0	0.01534	0.13804
10.0	0.01206	0.12061
20.0	0.00199	0.03971
30.0	0.00058	0.01733
40.0	---	0.00888
80.0	---	0.00135

*At the mode, $\hat{t} = 0.5584$, $g(\hat{t}) = 0.34470$ and $g(\hat{t}') = 0.19247$.
At the median, $\check{t} = 1.932$, $g(\check{t}) = 0.18530$ and $g(\check{t}') = 0.35800$.

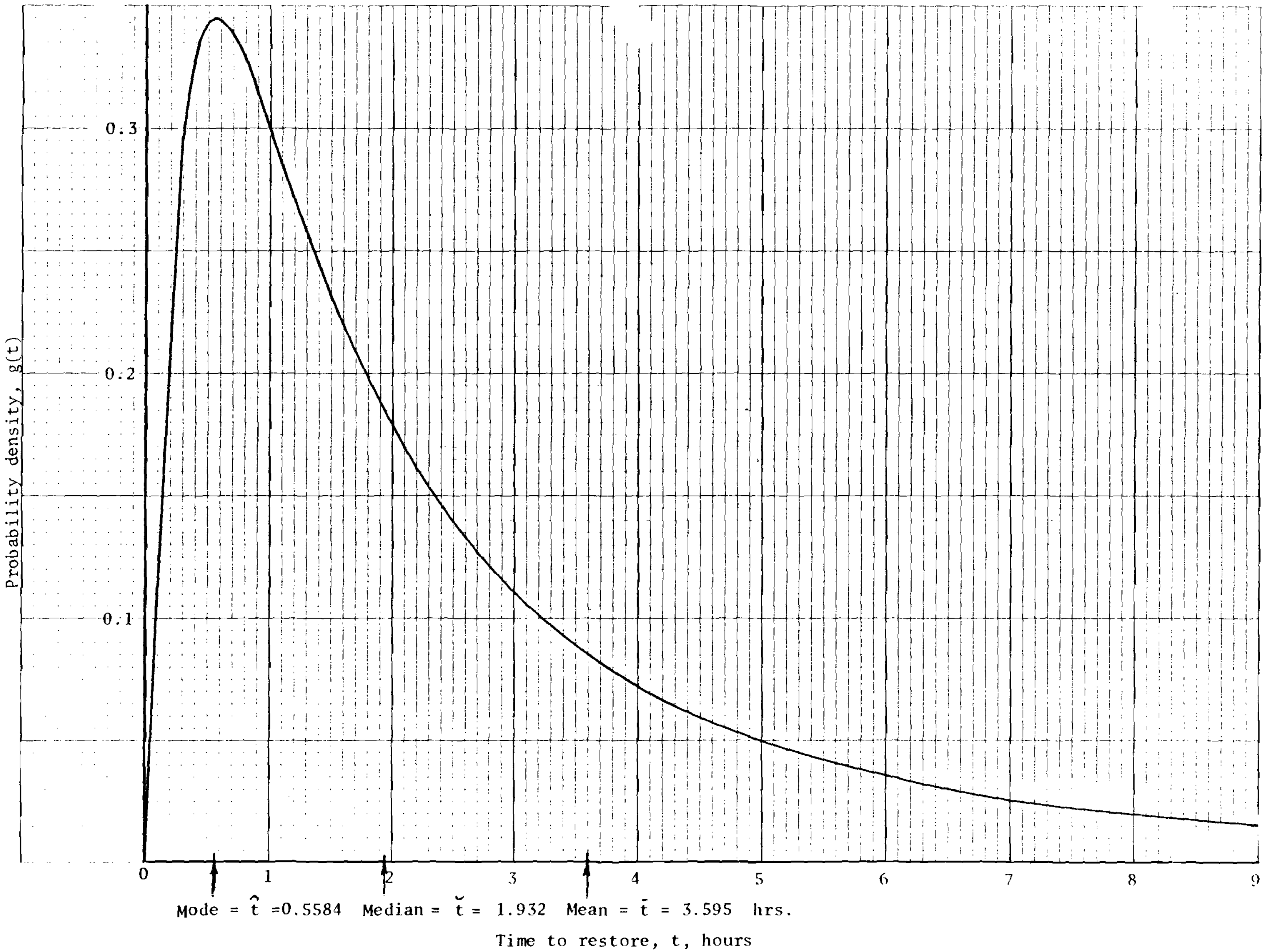

FIGURE 5.3.2.1.1-1: PLOT OF THE LOGNORMAL PDF OF THE TIMES-TO-RESTORE DATA GIVEN IN TABLE 5.3.2.1.1-3 IN TERMS OF THE STRAIGHT t's

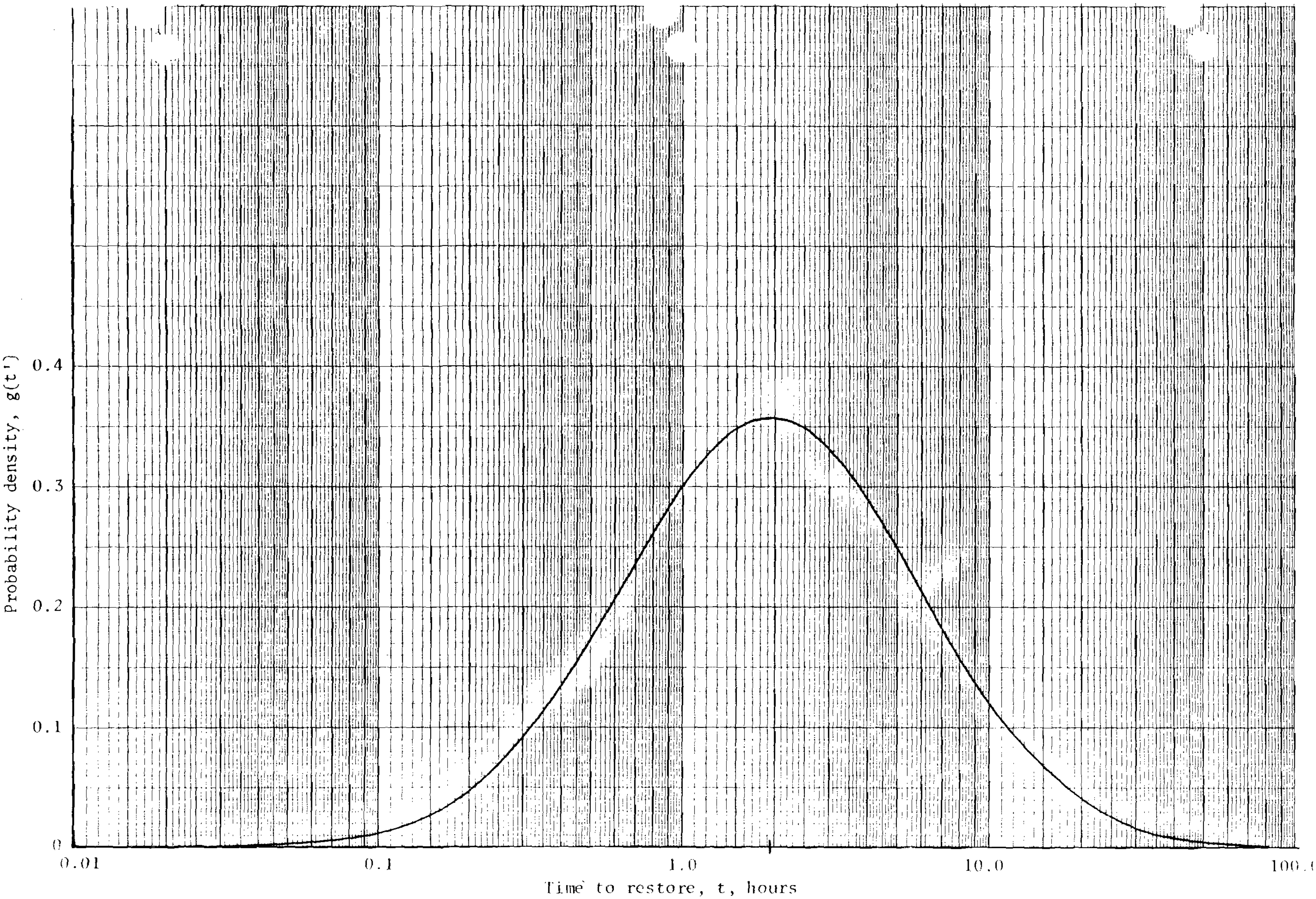

FIGURE 5.2.2.1.1-2: PLOT OF THE LOGNORMAL PDF OF THE TIMES-TO-RESTORE DATA GIVEN IN TABLE 5.3.2.1.1-3 IN TERMS OF THE LOGARITHMS OF t, OR $\mathrm{LOG}_e t = t'$

2. MTTR (Mean Time to Repair) of the System

The mean time to repair of the system, $\bar{t}$, is obtained from Eq. (5.73).

$$t = e^{(t' + 1/2 (\sigma_{t'})^2)}$$

$$t = e^{(0.65879 + 1/2 (1.11435)^2)}$$

or

$$\bar{t} = 3.595 \text{ hr.}$$

3. Median Time to Repair

The median of the times-to-repair the system, $\breve{t}$, is obtained from Eq. (5.76)

$$\breve{t} = e^{\bar{t}'}$$

$$\breve{t} = e^{0.65879}$$

or

$$\breve{t} = 1.932 \text{ hr.}$$

This means that in a large sample of t's half of the t's will have values smaller than $\breve{t}$, and the other half will have values greater than $\breve{t}$. In other words, 50% of the repair times will be $\leq \breve{t}$.

4. Maintainability Function M(t)

The maintainability of a unit can be evaluated as follows, using Eq. (5.62):

$$M(t_1) = \int_0^{t_1} g(t)\ dt = \int_{-\infty}^{t'_1} g(t')\ dt' = \int_{-\infty}^{z(t'_1)} \phi(z)\ dz \quad (5.81)$$

where $t' = \log_e t$, (5.81a)

$$z(t'_1) = \frac{t'_1 - \bar{t}'}{\sigma_{t'}} \quad (5.81b)$$

and $\bar{t}'$ and $\sigma_{t'}$ are given by Eq. (5.79) and (5.80), respectively.

By means of the transformations shown in Eqs. (5.81a) and (5.81b), the lognormal distribution of the pdf of repair times, g(t), is transformed to the standard normal distribution ϕ (z) which enables the use of standard normal distribution tables (Table A-1, Appendix A).

The maintainability function for the system, M(t), from (5.81) is:

$$M(t) = \int_{-\infty}^{z(t')} \phi(z)\, dz$$

where

$$z(t') = \frac{t' - \bar{t}'}{\sigma_{t'}}$$

$$t' = \log_e t$$

From the data in Table 5.3.2.1.1-1 we previously calculated

$$\bar{t}' = 0.65879$$

$$\sigma_{t'} = 1.11435$$

The quantified M(t) is shown in Figure 5.3.2.1.1-3. The values were obtained by inserting values for, $t' = \log_e t$, into the expression,

$$z(t') = \frac{t' - 0.65879}{1.11435}$$

solving for z(t'), and reading the value of M(t) directly from the standard normal tables in Appendix A (Table A-1).

5. Maintainability for a 20 Hour Mission

$$M(20) = \int_{-\infty}^{z(\log_e 20)} \phi(z)\, dz$$

where $\log_e 20 = 2.9957$
and

$$Z(\log_e 20) = \frac{2.9957 - 0.65879}{1.11435} = 2.0972$$

From Appendix A we find that for z = 2.0972

$$M(20) = \int_{-\infty}^{2.0972} \phi(Z)\, (dZ) = 1\text{-}0.018 = 0.982 \text{ or } 98.2\%$$

6. The time within which 90% and 95% of the Maintenance Actions are Completed ($M_{max_{ct}}$)

This is the time $t_{1-\alpha}$ for which the maintainability is $1-\alpha$, or

$$M(t_{1-\alpha}) = P(t \leq t_{1-\alpha}) = \int_0^{t_{1-\alpha}} g(t)\, dt = \int_{-\infty}^{t'_{1-\alpha}} g(t')\, dt' = \int_{-\infty}^{z(t'_{1-\alpha})} \phi(z)\, dz, \quad (5.82)$$

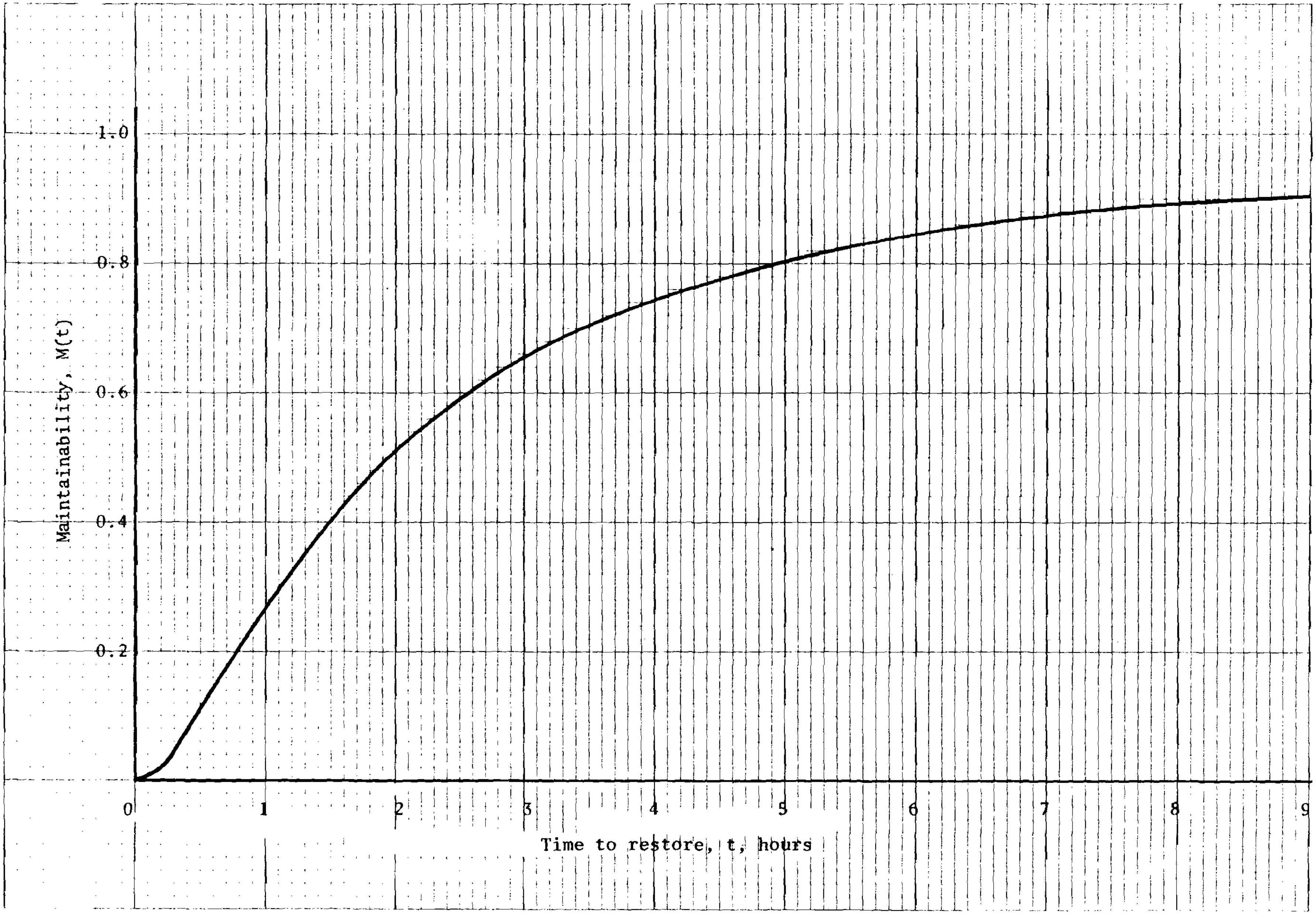

FIGURE 5.3.2.1.1-3: PLOT OF THE MAINTAINABILITY FUNCTION FOR THE TIMES-TO-REPAIR DATA OF EXAMPLE 2

and

$$z(t'_{1-\alpha}) = \frac{t'_{1-\alpha} - \bar{t}'}{\sigma\ t'} \tag{5.83}$$

The commonly used maintainability, or $(1-\alpha)$, values are 0.80, 0.85, 0.90, 0.95, and 0.99. Consequently, the $z(t'_{1-\alpha})$ values which would be used most commonly would be those previously given in Table 5.3.2.1-1. Using Eq. (5.83) the time $t'_{1-\alpha}$ would then be calculated from

$$t'_{1-\alpha} = \bar{t}' + z(t'_{1-\alpha}) \cdot \sigma_{t'}$$

or

$$t_{1-\alpha} = \text{antilog}_e(t'_{1-\alpha}) = \text{antilog}_e\left[\bar{t}' + z(t'_{1-\alpha}) \cdot \sigma_{t'}\right] \tag{5.84}$$

Thus, for 90% $M_{max_{ct}}$, from the previously obtained value of $\bar{t}'$ and σ_t

$$t_{0.90} = \text{antilog}_e\left[\bar{t}' + z\ (t'_{0.90})\sigma\ t'\right]$$

$$= \text{antilog}_e\left[0.65879 + 1.282\ (1.11435)\right]$$

$$= \text{antilog}_e\ (2.08737)$$

$$= 8.06 \text{ hrs.}$$

For 95% $M_{max_{ct}}$

$$t_{0.95} = \text{antilog}_e\left[0.65879 + 1.645\ (1.11435)\right]$$

$$= \text{antilog}_e\ (2.491896) = 12.08 \text{ hrs.}$$

7. <u>Repair Rate at t = 20 hours</u>

Using Eq. (5.63) and substituting the values for g(20) from Table 5.3.2.1.1-3 and the previously calculated value for M(20)

$$u(20) = \frac{g(20)}{1-M(20)} = \frac{0.00199}{1-0.982} = \frac{0.00199}{0.018}$$

$$= 0.11 \text{ repairs/hr.}$$